Development of a Full Car Vehicle Dynamics Model for Use in the Design of an Active Suspension Control System

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1. Introduction

This paper discusses the creation of a full car model for a standard road going vehicle. This model has been equipped with suspension force actuators to allow for the future development of an active suspension control system to improve the vehicle’s ride comfort. These types of systems are becoming increasingly common on both passenger and commercial vehicles. The flexibility these systems offer allows them to be specifically tuned for performance or comfort, making them optimum for many applications.

Active suspension is concerned with controlling the vertical movements of the vehicle in response to the road inputs to each of the wheels. This is accomplished by actively applying vertical forces in the suspension to counteract some of the effects of the road surface. As a result, these systems can be used to minimize vehicle body roll, vertical accelerations experienced by the passengers, and improve overall vehicle handling.

2. System Description

The vehicle model includes a suspension unit at each corner of the vehicle which consists of a spring, damper and a force actuator as shown in Fig.1. The constitutive behavior of these elements are non-linear and the linearization process is discussed in section 3.2.

The vehicle chassis is modeled as a rigid body with body fixed coordinates, U,V,W attached at the Center of Gravity (CG) and aligned in its principal directions as shown in Fig. 2. The body has mass, m, and moments of inertia J_r (roll) about the U-axis, J_p (pitch) about the V-axis, and J_y (yaw) about the W-axis. The CG is located a distance ‘a’ from the front axle, ‘b’ from the rear axle, and ‘h’ from the ground. The half-width of the vehicle is w/2.

The suspension actuators are implemented simply as controllable force inputs. The physical method of force actuation is not discussed in this paper. This will allow more flexibility once the control system has been designed for selecting the most appropriate actuator. The force actuation can be accomplished using a variety of components. Some examples include electromechanical actuators, hydraulic actuators, and pneumatic actuators. Each system has its own distinct strengths such as response time and power requirements.
2.1 Parameter Definition

2.1.1. Physical Parameters

- \( \delta \) steering angle (rad)
- \( a \) distance from CG to front axle (m)
- \( b \) distance from CG to rear axle (m)
- \( b_{SF} \) damper coefficient – front (N-s/m)
- \( b_{SR} \) damper coefficient – rear (N-s/m)
- \( F_{\text{brake}} \) total braking force (N)
- \( F_{\text{pitch}} \) pitching force on CG (N)
- \( F_{\text{roll}} \) rolling force on CG (N)
- \( F_{FR} \) controlled actuator output front right (N)
- \( F_{FL} \) controlled actuator output front left (N)
- \( F_{RR} \) controlled actuator output rear right (N)
- \( F_{RL} \) controlled actuator output rear left (N)
- \( h \) height of CG from road (m)
- \( J_p \) pitch moment of inertia (kg·m²)
- \( J_r \) roll moment of inertia (kg·m²)
- \( k_t \) tire stiffness (N/m)
- \( k_s \) spring stiffness (N/m)
- \( K_{RF} \) front anti-roll bar stiffness (N/m)
- \( K_{RR} \) rear anti-roll bar stiffness (N/m)
- \( m_s \) mass of the car (kg)
- \( m_{us} \) unsprung mass (kg)
- \( U \) forward velocity (m/s)
- \( V_{CG(t)} \) vertical velocity of CG (m/s)
- \( V_{FR(t)} \) velocity input front right (m/s)
- \( V_{FL(t)} \) velocity input front left (m/s)
- \( V_{RR(t)} \) velocity input rear right (m/s)
- \( V_{RL(t)} \) velocity input rear left (m/s)
- \( w \) track width (m)

2.1.2. State Variables Definition

- \( L_R \) rolling angular momentum (N·m·s)
- \( L_p \) pitching angular momentum (N·m·s)
- \( p_{usRR} \) unsprung momentum rear right (kg·m/s)
- \( p_{usRL} \) unsprung momentum rear left (kg·m/s)
- \( p_{usFR} \) unsprung momentum front right (kg·m/s)
- \( p_{usFL} \) unsprung momentum front left (kg·m/s)
- \( p_{CG} \) vertical momentum of CG (kg·m/s)
- \( x_{TR} \) tire deflection rear right (m)
- \( x_{TL} \) tire deflection rear left (m)
- \( x_{FR} \) tire deflection front right (m)
- \( x_{FL} \) tire deflection front left (m)
- \( x_{SRR} \) suspension spring deflection rear right (m)
- \( x_{SRL} \) suspension spring deflection rear left (m)
- \( x_{SFR} \) suspension spring deflection front right (m)
- \( x_{SFL} \) suspension spring deflection front left (m)
- \( x_{RB} \) anti-roll bar deflection rear (m)
- \( x_{RB} \) anti-roll bar deflection front (m)

2.2. Inputs

The system has ten inputs, six of which are exogenous and the others controllable. These inputs are:

Exogenous:
- The road velocity inputs experienced at each wheel
- Vehicle pitch force (due to accelerating/braking/cornering the vehicle)
- Vehicle roll input (due to cornering the vehicle)

Controllable:
- Actuator forces applied to the suspension system at each corner of the vehicle

In this simulation, the road inputs, vehicle pitch, and roll will be simulated based on three different driving scenarios:

1. Driving over a “speed bump” by generating a vertical velocity profile input
2. Braking at 1 g by applying the appropriate pitch moment to the vehicle center of gravity
3. Cornering by applying the appropriate pitch and roll moment to the vehicle center of gravity

2.3. Outputs

The model used in this simulation is composed of 17 separate state variables, however not all of these states are relevant to the control of an active suspension system.

The ride quality can be quantified by examining the vertical and angular accelerations of the vehicle body, as well as the ability for the vehicle to remain level regardless of operating conditions. [6]

The 17 states in this model each correspond to the state of an energy storing element. The following states are observed using the C matrix:

- The deflection of the suspension springs
- The deflection of the tire springs
- The vertical and angular velocities of the vehicle’s center of gravity
2.4 Noise

Road going automobiles are host to a plethora of electronic noise. This noise can be developed on board from the vehicle’s electronic ignition system, or off board from nearby power transmission lines. Noise was neglected in the development of the vehicle model, but will be accounted for in the development of an observer/estimator architecture of the force actuator control system (next paper).

3. System Model

3.1. Modeling Methodology

Modeling the aforementioned system began with the creation of a 'bond graph' of the system. Bond graphs are a concise pictorial representation of all types of interacting energy domains, and are an excellent tool for representing vehicle dynamics with associated control hardware[1]. Each bond represents a pair of signals (effort and flow) whose product is the instantaneous power of the bond. In the case of a mechanical system, effort and flow translate into force and velocity respectively. The 'half arrow' sign convention defines the direction of energy flow. The energy storing elements in the bond graph define the number of state variables in the system and using the established methods in bond graphing, state equations can be derived directly from the bond graph [4].

![Fig. 3 : Schematic of a single suspension unit and the corresponding bond graph [1]](image)

For illustration purpose, the schematic and the corresponding bond graph for a single suspension unit is shown in Fig. 3. Please refer to Appendix A for the complete bond graph of the system.

The set of state equations were derived using the complete bond graph as further discussed in section 3.3. State-space matrices (A,B,C,D) were derived using these system equations and are discussed further in section 3.4.

3.1.2. Underlying Assumptions

The objective of the model created was to assist in the development of a control system for the vehicle’s active suspension system. A high level of detail could have been included in the development of the model, however assumptions were made to simplify the model. These simplifications help remove unnecessary details that are not of interest when optimizing the vertical dynamics of the vehicle. The assumptions also help reduce the computational requirements of the simulation.

The following assumptions were made to simplify the model:

- The body of the vehicle is rigid.
- The lateral and longitudinal motion of the tires is negligible compared to their vertical motion.
- The vehicle is a neutral steer car[1,2]
- The vehicle is not skidding

Because lateral and longitudinal dynamics have been removed from the model, it was important to approximate their effects on the vertical behavior of the model during braking/cornering. The approximated braking/cornering forces are applied to the CG of the vehicle, as discussed in more detail in the Simulation section of the paper (Section 4.3-4.4).

3.2 Linearization

A strength of bond graph modeling is the ability to use a single model for both linear and nonlinear systems over multiple energy domains. The ordinary differential equations that describe the system are extracted directly from the bond graph using a straightforward procedure. Each component has particular constitutive laws that describe its behavior and are tied together at the time of equation formulation. The switch from a nonlinear to a linear component comes from a simple substitution in the bond graph equations.

The modeled components are in reality nonlinear; however a standard linearization process can be executed for each component. Fig. 4 shows a hypothetical tire deflection curve in red. A tire is unable to “pull” (provide negative force) since it is not attached to the ground. In addition, the positive force that it supplies is nonlinear. To linearize this tire, the equilibrium point on the actual curve must be located. For small deviations from the equilibrium point, the constitutive behavior of the spring may be
considered linear as shown in blue. The linear tire is a particularly complicated component due to its inability to prevent the application of negative force. This must be dealt with by adding logic into the simulation code, or by scaling the inputs to prevent tire lift-off.

The suspension springs, and dampers would typically undergo a similar linearization process. This particular model however is based loosely on actual vehicle data of a mid size sedan as mentioned in reference [1], and the original equations that were linearized to provide the constants tabulated in table 1 were unavailable.[1]

The following assumptions about linearity were made in our model:

- Each tire is modeled as a single linear spring
- Each of the suspension springs are linear
- Every linear spring element (tire and suspension) has an equilibrium displacement calculated by the static vehicle model sitting in a gravity acceleration field
- Each of the suspension dampers are linear
- Each of the active suspension force actuators are linear

The linearity of this model permits the use of a state-space representation of the system. This results in first-order explicit differential equations of the form

\[ \dot{X} = AX + BU \]  

that are easily numerically integrated.

### 3.3 State Variables & Linearized System Equations

As discussed previously in section 3.1, using the bond graph, 17 state variables were identified and linear state equations were obtained. Please refer to Appendix B for the complete set of state equations. For more information on the procedure of deriving state equations using bond graphs, please refer to reference [4].

### 3.4 State-Space Representation

Please refer to Appendix C for the complete set of state space representation matrices obtained from the linearized state equations.

### 3.5 Controllability, Observability and Stability

Controllability was observed by determining the rank of the controllability matrix

\[ \begin{bmatrix} B & BA & BA^2 & \ldots & BA^{n-1} \end{bmatrix} \]

where B and A are the state space matrices as shown in Appendix C and n is the number of states; 17 for this model. It was observed that there are 9 uncontrollable states in the system using the function ctrb() in Matlab. Bond graphs also offer a method to identify these uncontrollable states by propagating the effect of each input through the graph. By doing so, it was discovered that all states of the system were controllable by the four force actuator inputs. This discrepancy between the Matlab result and the intuitive result are discussed below.

Similarly, the Observability matrix

\[ \begin{bmatrix} C & CA & CA^2 & \ldots & CA^{n-1} \end{bmatrix}^T \]

was analyzed in Matlab using the C and A state space matrices (shown in Appendix C) and it was found that 8 states were unobservable. Observability was also examined intuitively through the system bond graph. It was discovered that every state of the system was observable via the Cg roll angle, pitch angle, and vertical position. These three outputs would be produced in a physical vehicle by the integration of an accelerometer signal produced at the vehicle’s CG. The discrepancy between the observability predicted by the observavility matrix rank, and the intuitive investigation of the model is discussed below.

Stability of the system was checked by determining the Eigen-values of the A matrix. All Eigen-values had a real component less than or equal to zero, and thus the system was deemed stable. Of the 17 Eigen-values there were 3 with a value of zero. These zero
values may explain the discrepancy between the Matlab derived controllability/observability results, and the intuitive results. One physical example of a zero mode of a rigid body vehicle is the following: displacing the left front and right rear suspension in the positive direction, and displacing the right front and left rear suspension by the same amount in the opposite direction. Releasing the vehicle from the said state will inflict no motion, and thus the mode frequency is said to be zero.

4. Simulation

Table 1 shows the linear parameters used for the vehicle simulation, which are loosely based on a standard sedan [1]. These parameters were used to populate the state-space representation of the model shown in Appendix C. A Simulink model was constructed which allowed inputs and outputs to be applied to/recorded from a state-space block. The Simulink block diagram and the state-space A,B,C,D matrix population code can be viewed in Appendix D and E respectively.

Table 1: Parameter Values for Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vehicle</strong></td>
<td></td>
</tr>
<tr>
<td>Distance from Cg to front axle (a)</td>
<td>1.17m</td>
</tr>
<tr>
<td>Distance from Cg to rear axle (b)</td>
<td>1.68m</td>
</tr>
<tr>
<td>Height of Cg above the road (h)</td>
<td>0.55m</td>
</tr>
<tr>
<td>Track (w)</td>
<td>1.54m</td>
</tr>
<tr>
<td>Mass of the car (m,)</td>
<td>1513 kg</td>
</tr>
<tr>
<td>Roll moment of inertia (J,)</td>
<td>637.26 kgm²</td>
</tr>
<tr>
<td>Pitch moment of inertia (Jr)</td>
<td>2443.26 kgm²</td>
</tr>
<tr>
<td>Anti-roll bar stiffness (kr)</td>
<td>1.5 x 10⁶ N/m</td>
</tr>
<tr>
<td><strong>Tire</strong></td>
<td></td>
</tr>
<tr>
<td>Unsprung mass (m,u)</td>
<td>38.42 kg</td>
</tr>
<tr>
<td>Tire stiffness (k_t)</td>
<td>150,000 N/m</td>
</tr>
<tr>
<td><strong>Suspension</strong></td>
<td></td>
</tr>
<tr>
<td>Suspension stiffness (k_s)</td>
<td>14,900 N/m</td>
</tr>
<tr>
<td>Damper coefficients (b_s)</td>
<td>475 Ns/m</td>
</tr>
</tbody>
</table>

4.1 Model Validation using a quarter-car

Before complicated full car simulations could be conducted, it was necessary to ensure that basic properties indicative of the quarter car model were evident in the simulation results. To simulate the model as a quarter car, the model was made geometrically symmetric by setting distance “a” equal to distance “b”, effectively placing the model’s center of gravity symmetrically between the front and rear of the vehicle. By inputting the same velocity at each corner of the vehicle, the body of the vehicle exhibited only vertical motion, represented by a simple mass-spring-damper system, as shown below in Fig. 5.

Analysis of this simple spring-mass-damper system produced Eqn. 2 and 3 which, when evaluated with the parameters listed in Table 1, resulted in a body natural frequency of 0.95Hz, and a wheel natural frequency of 10.4Hz. [2]

\[
\begin{align*}
\omega_{body} &= \frac{1}{2\pi} \sqrt{\frac{ks-k_t}{(ks+k_t)m_u/k}} = 0.95Hz \\
\omega_{wheel} &= \frac{1}{2\pi} \sqrt{\frac{ks+k_t}{m_{ax}}}=10.4Hz
\end{align*}
\]

The vehicle model was given a velocity step input of 5 m/s at each corner, lasting for 0.2 seconds. The abrupt application of velocity excited the faster wheel hop frequency, which was quickly damped giving way to the slower body oscillations as seen below in Fig. 6. A Fast Fourier Transform was applied to the resulting suspension displacement data, and the dominant frequency was found to be 0.95Hz which correlates with the anticipated value calculated in Eqn. 2.

![Fig. 5: Quarter Car Model](image)

![Fig. 6: Body Natural Frequency FFT](image)
To focus on recording the wheel natural frequency, the Fast Fourier Transform was concentrated on the first quarter second of the simulation when fast oscillations were prevalent. The result, shown below in Fig. 7 is in agreement with the frequency predicted by Eqn. 3. The full vehicle model behaved as expected when simulated as a quarter car.

![Fig. 7: Wheel Hop natural Frequency FFT](image)

4.2 Scenario 1 – Road Irregularity

To evaluate the vehicle’s performance over road irregularities, a triangular profile speed bump was constructed with a width of 20cm, and a height of 5cm. The vehicle was simulated driving over the bump at 5m/s (18 km/h), producing the tire displacement plot shown in Fig. 8. The front of the vehicle encounters the bump first, with the rear of the vehicle shortly following.

![Fig. 8: Tire Displacement Driving Over Speed-Bump at 5m/s](image)

As described in the section 3.2, the linearized tire may produce force in both compression and tension, while a true tire may only produce force while compressed. Driving over a bump too quickly with a physical vehicle will cause the tires to leave the ground momentarily at the exit of the bump. Fig. 8 shows the displacement of the front and rear tires of the vehicle from equilibrium. As it can be observed, the front tires of the vehicle remain in contact with the ground. Unfortunately, the rear tires of the vehicle lift off the ground shortly after encountering the bump. This behavior must be detected and avoided as the linear tire model used here does not account for such situations.

4.3 Scenario 2 - Braking

To understand how the uncontrolled model reacted to braking, a step input equivalent to the force of braking at 1.0 g (calculated using Eqn. 4 was applied to the pitch axis of the vehicle.

\[
F_{\text{brake}} = (m_{\text{us}} + m_{\text{us}}) \times a \quad (4)
\]

\[
F_{\text{brake}} = (1513 \text{kg} + 4 \times 38.42 \text{kg}) \times \frac{9.81 \text{m}}{\text{s}^2} \\
= 16350 \text{N}
\]

The resulting pitch angle, and angular acceleration of the vehicle about its center of gravity are show below in Fig. 9. During the transient period following the application of the force, the vehicle experiences oscillating angular accelerations about the pitch axis which would be uncomfortable to the occupants of the vehicle.

![Fig. 9: 1.0 g Braking](image)
These oscillations are likely exaggerated due to the instantaneous nature of a step input, and would likely be reduced during a physical braking test. These oscillations are mostly damped out of the system within 5 seconds, and the vehicle is left with a steady state pitch angle of 3.3 degrees. The application of a control system will have two objectives during braking. Firstly, it will work to reduce the angular acceleration experienced due to the application of brakes. Secondly, it will keep the vehicle as level as possible, minimizing the steady state angle produced from the braking force.

4.4 Scenario 3 - Constant Radius Turn

Unlike braking, cornering produces a moment about the roll and pitch axis. This is due to the fact that tire forces act approximately perpendicular to the plane of the tire. Fig. 10 shows the creation of transverse forces due to the steering angle, delta.

The vehicle was simulated as completing a 0.5 g corner with a forward velocity of 22.2 m/s (80 km/hr). Neglecting the tire slip angles, the required steering angle, delta, can be approximated using Eqn. 5.

$$
\delta = \frac{a_{LONG} \cdot \frac{a+b}{u^2}}{2} \quad (5)
$$

$$
\delta = 4.9 \left( \frac{m}{s^2} \right) \cdot \frac{1.17 + 1.68 m}{54 (s^2)} = 0.5586 \text{ rad}
$$

The transverse and longitudinal forces, which will be applied to the pitch and roll axis of the vehicle’s center of gravity were then calculated using Eqn. 6 and 7.

$$
F_{Long} = m_{total} \cdot a_{corner} = 1667 kg \cdot 4.9 \left( \frac{m}{s^2} \right)
= 8168 N
$$

$$
F_{Trans} = \frac{F_{Long} \cdot \sin (\delta)}{1 + \cos (\delta)} = \frac{8168 N \cdot \sin (0.5586)}{1 + \cos (0.5586)}
= 2342 N
$$

Unlike braking, cornering inputs were ramped up to the maximum desired value, then held constant. This was done to approximate the slow increase in steering angle at the corner entrance, and then the steady steering angle for steady state cornering. Fig. 11 shows the results of the simulated corner. During steady state cornering at 0.5 g the body of the vehicle exhibits a roll angle of almost 1.5 degrees which the future control system will look to minimize. The angular acceleration, peaking at about 0.2 rad/s², is very small due to the ramped steering input, as compared to almost 4.0 m/s² observed in the step input braking study.

To understand if the model was correctly approximating the behavior of a physical vehicle, suspension displacement plots were produced for the steered vehicle as seen below in Fig. 12.

The front right suspension system is compressed by approximately 3.3 cm, and the right rear suspension system is unloaded by approximately the same amount. The left front and right rear suspension components exhibit displacements an order of magnitude less than their diagonal counterparts.
These data show that weight was transferred diagonally from the left rear of the vehicle, to the right front of the vehicle. This is the expected outcome of a physical vehicle which is making a left turn. It is also important to note that the suspension displacement of 3.3cm is realistic for a road going vehicle, showing that the parameter values used in table 1 are reasonable. The negative value for the left rear suspension displacement represents unloading of the suspension, as a displacement of 0 m signifies the equilibrium position of the suspension system due to the vehicle sitting in a 1 g gravity field.

5. Conclusion

A linear vehicle dynamics model has been constructed which focuses on the vertical motion of a vehicle due to road irregularities. This model avoids the use of complicated lateral/longitudinal vehicle dynamics, and instead approximated their application to the CG of the vehicle.

The model has been validated in quarter car, and full car simulations. The model successfully approximated bump, brake, and cornering situations.

Future work will be to improve passenger ride comfort by implementing an active suspension control system. This improvement will be accomplished through the utilization of force actuators tied into each corner of the vehicle’s suspension system.

6. References

Appendix List

A : Full Bond Graphs
B : State Equations
C : State Space Representation : A, B, C, D matrices
D : Complete Simulink Model
E : MATLAB code
Appendix B : State Equations

Tire Springs

\[
\begin{align*}
\dot{x}_{TRR} &= V_{RR}(t) - \frac{p_{usRR}}{m_{us}} \\
\dot{x}_{TRL} &= V_{RL}(t) - \frac{p_{usRL}}{m_{us}} \\
\dot{x}_{TFR} &= V_{FL}(t) - \frac{p_{usFR}}{m_{us}} \\
\dot{x}_{TFL} &= V_{FR}(t) - \frac{p_{usFL}}{m_{us}}
\end{align*}
\]

Anti-roll bars

\[
\begin{align*}
\dot{x}_{RBR} &= w \left( \frac{L_R}{f_r} \right) \\
\dot{x}_{RBF} &= w \left( \frac{L_R}{f_r} \right)
\end{align*}
\]

Unsprung mass momentum

\[
\begin{align*}
\dot{p}_{usRR} &= \frac{x_{TRR}}{k_t} - \left[ -F_{RR} - \frac{x_{SRR}}{k_s} + b_{SR} \left( \frac{p_{usRR}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \frac{L_p}{J_p} \right) + \frac{w}{2} \right] \\
\dot{p}_{usRL} &= \frac{x_{TRL}}{k_t} - \left[ -F_{RL} - \frac{x_{SRL}}{k_s} + b_{SR} \left( \frac{p_{usRL}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \frac{L_p}{J_p} \right) - \frac{w}{2} \right] \\
\dot{p}_{usFR} &= \frac{x_{TFR}}{k_t} - \left[ -F_{FR} - \frac{x_{SFR}}{k_s} + b_{SF} \left( \frac{p_{usFR}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \frac{L_p}{J_p} \right) + \frac{w}{2} \right] \\
\dot{p}_{usFL} &= \frac{x_{TFL}}{k_t} - \left[ -F_{FL} - \frac{x_{SFL}}{k_s} + b_{SF} \left( \frac{p_{usFL}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \frac{L_p}{J_p} \right) - \frac{w}{2} \right]
\end{align*}
\]
Rate of suspension spring deflections

\[ \dot{x}_{SRR} = \frac{p_{usRR}}{m_{us}} - \left[ \frac{p_{VCG}}{m_s} + b \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \]

\[ \dot{x}_{SRL} = \frac{p_{usRL}}{m_{us}} - \left[ \frac{p_{VCG}}{m_s} - a \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \]

\[ \dot{x}_{SFR} = \frac{p_{usFR}}{m_{us}} - \left[ \frac{p_{VCG}}{m_s} + b \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \]

\[ \dot{x}_{SFL} = \frac{p_{usFL}}{m_{us}} - \left[ \frac{p_{VCG}}{m_s} - a \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \]

Vertical Momentum of Cg

\[ \dot{p}_{VCG} = -F_{FR} \frac{x_{SRR}}{k_s} + b_{SF} \left[ \frac{p_{usFR}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] - F_{RR} + \frac{x_{SRR}}{k_s} \]

\[ + b_{SF} \left[ \frac{p_{usRR}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] - F_{RL} + \frac{x_{SRL}}{k_s} \]

\[ + b_{SR} \left[ \frac{p_{usRL}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] - F_{FL} + \frac{x_{SFL}}{k_s} \]

\[ + b_{SR} \left[ \frac{p_{usFR}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \]

\[ L_p = h(F_{pitch}) + a \left[ \frac{x_{RBF}}{K_{RF}} + F_{FR} - \frac{x_{SFR}}{k_s} - b_{SF} \left[ \frac{p_{usFR}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \right] \]

\[ + a \left[ \frac{x_{RBF}}{K_{RF}} + F_{FL} - \frac{x_{SFL}}{k_s} - b_{SR} \left[ \frac{p_{usFL}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \right] \]

\[ + b \left[ \frac{x_{RBR}}{K_{RR}} - F_{RR} + \frac{x_{SRR}}{k_s} + b_{SF} \left[ \frac{p_{usRR}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \right] \]

\[ + b \left[ \frac{x_{RBR}}{K_{RR}} - F_{RL} - \frac{x_{SRL}}{k_s} + b_{SR} \left[ \frac{p_{usRL}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_R} \right) \right] \right] \]
\( \dot{L}_R = h(F_{roll}) - \frac{w}{2} \left[ \frac{x_{RBR}}{K_{RF}} - F_{FR} + \frac{x_{SFR}}{k_s} + b_{SF} \left\{ \frac{p_{32}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_r} \right) \right\} \right] \\
- \frac{w}{2} \left[ \frac{x_{RBR}}{K_{RF}} + F_{FL} - \frac{x_{SFL}}{k_s} - b_{SF} \left\{ \frac{p_{32}}{m_{us}} - \frac{p_{VCG}}{m_s} + a \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_r} \right) \right\} \right] \\
- \frac{w}{2} \left[ \frac{x_{RBF}}{K_{RR}} - F_{RR} + \frac{x_{SRB}}{k_s} + b_{SR} \left\{ \frac{p_{32}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \left( \frac{L_p}{J_p} \right) + \frac{w}{2} \left( \frac{L_R}{J_r} \right) \right\} \right] \\
- \frac{w}{2} \left[ -\frac{x_{RBF}}{K_{RR}} - F_{RL} - \frac{x_{SRL}}{k_s} + b_{SR} \left\{ \frac{p_{32}}{m_{us}} - \frac{p_{VCG}}{m_s} - b \left( \frac{L_p}{J_p} \right) - \frac{w}{2} \left( \frac{L_R}{J_r} \right) \right\} \right] 
\)

Appendix C : State Space Representation : A, B, C, D matrices

\[
X = \begin{bmatrix} x_{TRL} \\ x_{TFR} \\ x_{SRL} \\ x_{TFL} \\ x_{TRR} \\ x_{SFR} \\ x_{RBR} \\ x_{SFL} \\ p_{usRL} \\ x_{SRR} \\ p_{usFR} \\ x_{RBR} \\ p_{usFL} \\ p_{usRR} \\ L_R \\ L_P \\ p_{VCG} \end{bmatrix}, \quad 
U = \begin{bmatrix} V_{RR} \\ F_{RR} \\ F\text{roll} \\ F_{RL} \\ V_{RL} \\ F_{FL} \\ F_{FR} \\ V_{FL} \end{bmatrix}, \quad 
B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad 
D_{(11x10)} = 0
\]
Please refer to the MATLAB code in Appendix E for the complete A matrix programmed in.

\[
A = \begin{bmatrix}
  0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{m_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{m_x} & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{m_x} & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
  0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Suspension Displacement:
1) LR
2) LF
3) RF
4) RR
5) CG Verticle Velocity
6) CG Pitch Angular Velocity
7) CF Roll Angular Velocity

Tire Displacement:
8) LR
9) LF
10) RF
11) RR
12) CG Verticle Acceleration
13) CG Verticle Displacement
14) CG Pitch Angle
15) CG Alpha Roll
16) CG Roll Angle
17) CG Alpha Pitch

Inputs:
1) Pitch Force
2) Roll Force
3) RR Velocity
4) RR Force Actuator
5) LR Force Actuator
6) LF Force Actuator
7) LF Velocity
8) Time
9) Front Bump Input Position
10) Rear Bump Input Position
11) Clock

Outputs:
1) Pitch Amp
2) Roll Amp
3) LL Verticle Acceleration
4) LL Verticle Displacement
5) LL Alpha Pitch
6) LL Alpha Roll
7) LL Roll Angle
8) LL Pitch Angle
9) Front Bump Velocity
10) Rear Bump Velocity

Appendix D: Complete Simulink Model
Appendix E: MATLAB code

```matlab
clear all; clc

%----State Vector----------------------------------------------------------
%... Number of States 17
% Q47- Qtlr = 1 ;
% Q52- Qtlf = 2 ;
% Q56- Qtrf = 3 ;
% Q72- Qtrr = 4 ;
% Q42- Qslr = 5 ;
% Q62- Qarbf = 6 ;
% Q67- Qslf = 7 ;
% Q56- Qarbr = 9 ;
% Q28- Qsrr = 10 ;
% P45- Puslr = 11 ;
% P70- Puslf = 12 ;
% P52- Pusrf = 13 ;
% P33- Pi = 14 ;
% P24- Pusrr = 15 ;
% P75- Pr = 16 ;
% P74- Pp = 17 ;

%----Geometric Parameters-------------------------------------------------
w=1.54; %m
h=0.55; %m
b=1.68; %m
a=1.17; %m

%----Parameter Values------------------------------------------------------
Ctrr=1/150000; %1/(N/m)
Ctlr=1/150000; %1/(N/m)
Ctlf=1/150000; %1/(N/m)
Ctrf=1/150000; %1/(N/m)
Csrr=1/14900; %1/(N/m)
Cslr=1/14900; %1/(N/m)
Cslf=1/14900; %1/(N/m)
Ccrr=1/14900; %1/(N/m)
Carbr=1/30000; %1/(N/m)
Carbf=1/30000; %1/(N/m)
I =1513; %kg
Ir=637.26; %kg-m^2
Ip=2443.26; %kg-m^2
Iusrr=38.42; %kg
Iuslr=38.42; %kg
Iuslf=38.42; %kg
Iusrf=38.42; %kg
Rdrr=475; %N-s/m
Rdlr=475; %N-s/m
Rdrlr=475; %N-s/m
Rdrl=475; %N-s/m

%----Tire Static Displacement Calculator-----------------------------------
g=9.81; %m/s^2
Ff=g*(Iuslf+(I/2)*(b/(a+b)))); %N
Fr=g*(Iuslr+(I/2)*(a/(a+b)))); %N
Xft=Ff*Ctlf; %m
Xrt=Fr*Ctlr; %m

%----Mapping Our Variable Names to Campg Assignments----------------------
T1x2 = 2/w ;
T3x4 = 1/b ;
T5x6 = w/2 ;
T7x8 = a ;
T9x10 = 1/b ;
T11x12 = a ;
```
\[ T_{13 \times 14} = \frac{2}{w} \; ; \]
\[ T_{15 \times 16} = \frac{w}{2} \; ; \]
\[ T_{17 \times 18} = \frac{1}{h} \; ; \]
\[ T_{19 \times 20} = \frac{1}{h} \; ; \]

\[ C_{22} = C_{\text{trr}} \; ; \]
\[ I_{24} = I_{\text{usr}} \; ; \]
\[ R_{27} = R_{\text{drr}} \; ; \]
\[ C_{28} = C_{\text{sr}} \; ; \]
\[ I_{33} = I \; ; \]
\[ C_{37} = C_{\text{br}} \; ; \]
\[ R_{41} = R_{\text{dl}} \; ; \]
\[ C_{42} = C_{\text{sl}} \; ; \]

%----Mapping Our in/out to Campg Assignments-------------------------------

% SE17 = Fpitch ;
% SE19 = Froll ;
% SF21 = Vrr ;
% SE43 = Flr ;
% SF48 = Vlr ;
% SF49 = Vrf ;
% SE57 = Flf ;
% SF58 = Pfl ;
% SF73 = Vlf ;

%----Building the A and B matrices---------------------------------------

\[
A(1,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/I_{45} & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(1,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix};
\]

\[
A(2,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(2,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(3,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(3,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(4,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(4,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(5,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(5,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(6,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(6,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(7,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(7,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(8,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(8,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(9,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(9,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(10,:) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(10,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(11,:) = \begin{bmatrix} +1/C_{47} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(11,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]

\[
A(12,:) = \begin{bmatrix} 0 & 1/C_{72} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix};
\]
\[
B(12,:) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix};
\]
A(13,:) = [0,0,1/C50,0,0,0,-1/C56,0,0,0,0,-1/I52*R55,+1/I33*R55,...
0,-1/I75*T15x16*R55,-1/I74*T7x8*R55];
B(13,:) = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0];
A(14,:) = [0,0,0,1/C42,+1/C62-1/C62,+1/C67,+1/C56,+1/C37-1/C37,+...
1/C28,+1/I45*R41,+1/I70*R66,+1/I52*R55,-1/I33*R41-...
1/I33*R55-1/I33*R66,1/I24*R27,+1/I75*T5x6*R27-1/I75*Tlx2*R41-...
1/I75*T15x16*R55-1/I75*T13x14*R66,-1/I74/T3x4*R27-1/I74/T9x10*R41+...
1/I74*T7x8*R55+1/I74*T11x12*R66];
B(14,:) = [0,0,0,0,0,-1,0,0,-1,0,0];
A(15,:) = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0];
B(15,:) = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0];
A(16,:) = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,...
1/C56*T15x16,-1/C37/Tlx2-1/C37*T5x6,-1/C28*T5x6,+1/I45*R41/Tlx2,+...
1/I70*R66/T13x14,-1/I52*R55*T5x6,-1/I33*R41/Tlx2+1/I33*R27*T5x6-...
1/I33*R66/T13x14+1/I33*R55*T5x6,-1/I24*R27+T5x6,-1/I75*Tlx2*R41/Tlx2-...
1/I75*T5x6*R27+T5x6-1/I75*T13x14*R66/T13x14-1/I74*T7x8*R55+1/I74*T11x12*R66/T13x14-...
1/I74*T7x8*R55+1/I74*T11x12*R66];
B(16,:) = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0];
A(17,:) = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,...
1/C56*T7x8,+1/C37/T3x4-1/C37/T9x10,+1/C28/T3x4,+1/I45*R41/T9x10,-...
1/I70*R66*T11x12,-1/I52*R55*T7x8,-1/I33*R27/T3x4+1/I33*R55*T7x8-...
1/I33*R41/T9x10+1/I33*R66*T11x12,1/I24*R27/T3x4,+1/I75*T5x6*R27/T3x4=...
1/I175*T15x16*R55*T7x8-1/I175/Tlx2*R41/T9x10+1/I175*T13x14*R66*T11x2,-...
1/I174*T3x4*R7x8+1/I174*T7x8*R55*T7x8-1/I174/T9x10*R41/T9x10-...
1/I174*T11x12*R66*T11x12];
B(17,:) = [+1/T17x18,0,0,-1/T3x4,-1/T9x10,0,0,1*T7x8,+1*T1x12,...
0];

%==========================================================================
%Output Matrix Definition
C=zeros(11,17);
C(1,5)=1; C(2,7)=1; C(3,8)=1; C(4,10)=1; C(5,14)=1/I; C(6,17)=1/IP;...
C(7,16)=1/IR; C(8,1)=1; C(9,2)=1; C(10,3)=1; C(11,4)=1;
D=zeros(11,10);

%%
%This cell grabs the simulation output, and allows for sophisticated
%plotting

%%%%Get Data from Workspace, Organise--------------------------------------
Output=Out.signals.values;
Input=In.signals.values;
Ifpitch=Input(:,1); Ifroll=Input(:,3); Ivrr=Input(:,4);...
Ifl=Input(:,5); Ivrl=Input(:,7); Ifrr=Input(:,8);...
Ifl=Input(:,9); Ivlf=Input(:,10); t=Input(:,11);...
Iffront=Input(:,12); Ifr=Input(:,13);

Oxlr=Output(:,1); Oxlf=Output(:,2); Oxrf=Output(:,3); Oxrr=Output(:,4);...
Ow=Output(:,5); Owp=Output(:,6); Or=Output(:,7);...
OvertA=Output(:,12); Oheight=Output(:,13); OalphP=Output(:,14);...
Olpitch=Output(:,15); OalphR=Output(:,16); Oroll=Output(:,17);...
Otlr=Output(:,8); Otlf=Output(:,9); Otrf=Output(:,10);...
Otrr=Output(:,11);

%%%%Insure thatdata has been collected from the input/output correctly----
if length(t)~=length(Oxlr)
display('Error in In/Out Vector Sized')
break
end
%----Plots---------------------------------------------------------------
figure('Name','Wheel Velocity Input')
subplot(2,2,3), plot(t,Ivlr); xlabel('Time (s)'); ylabel('Vlr (m/s)')
subplot(2,2,4), plot(t,Ivrr); xlabel('Time (s)'); ylabel('Vrr (m/s)')
subplot(2,2,1), plot(t,Ivlf); xlabel('Time (s)'); ylabel('Vlf (m/s)')
subplot(2,2,2), plot(t,Ivrf); xlabel('Time (s)'); ylabel('Vrf (m/s)')

figure('Name','Wheel Position Output')
subplot(2,2,3), plot(t,Oxlr); xlabel('Time (s)'); ylabel('Xlr (m)')
subplot(2,2,4), plot(t,Oxrr); xlabel('Time (s)'); ylabel('Xrr (m)')
subplot(2,2,1), plot(t,Oxlf); xlabel('Time (s)'); ylabel('Xlf (m)')
subplot(2,2,2), plot(t,Oxrf); xlabel('Time (s)'); ylabel('Xrf (m)')
dummy=0:0.001:t(length(t));
figure('Name','Tire Position Output')
subplot(2,1,2), plot(t,Otlr-Irear,t,Irear,'k',dummy,Xrt,'r');
xlabel('Time (s)'); ylabel('Rear Tire Disp (m)');
legend('Tire Deflection','Input Bump Displacement',...'
'Loss of Tire Contact with Ground')
subplot(2,1,1), plot(t,Otlf-Ifront,t,Ifront,'k',dummy,Xft,'r');
xlabel('Time (s)'); ylabel('Front Tire Disp (m)');
legend('Tire Deflection','Input Bump Displacement',...'
'Loss of Tire Contact with Ground')
figure('Name','Roll Angle')
subplot(2,1,1)
hl1 = line(t,Ifroll,'Color','r');
ax1 = gca;
set(ax1,'XColor','k','YColor','r')
xlabel('Time (s)'); ylabel('Roll Force (N)');
ax2 = axes('Position',get(ax1,'Position'),...'
''XAxisLocation','top','YAxisLocation','right',...'
''Color','none','XColor','k','YColor','b');
hl2 = line(t,Oroll*180/pi,'Color','b','Parent',ax2);
ylabel('Roll Angle (degrees)')
subplot(2,1,2), plot(t,OalphR)
xlabel('Time (s)'); ylabel('Roll Axis Angular Acceleration (rad/s^2)')
figure('Name','Pitch Angle')
subplot(2,1,1),
hl1 = line(t,Ifpitch,'Color','r');
ax1 = gca;
set(ax1,'XColor','k','YColor','r')
xlabel('Time (s)'); ylabel('Pitch Force (N)');
ax2 = axes('Position',get(ax1,'Position'),...'
''XAxisLocation','top','YAxisLocation','right',...'
''Color','none','XColor','k','YColor','b');
hl2 = line(t,Opitch*180/pi,'Color','b','Parent',ax2);
ylabel('Pitch Angle (degrees)')
subplot(2,1,2), plot(t,OalphP)
xlabel('Time (s)'); ylabel('Pitch Axis Angular Acceleration (rad/s^2)')
figure('Name','CG Verticle Velocity')
plot(t,Ow,'b');
legend('CG Verticle Velocity'); xlabel('Time (s)'); ylabel('Velocity (m/s)')
figure('Name','Bump Position Plot')
subplot(2,1,1),plot(t,Ifront,'r',t,Oxrf,'b')
xlabel('Time(s)'); ylabel('Position (m)')
legend('Front Bump Input','Front Suspension Displacement')
subplot(2,1,2),plot(t,Irear,'r',t,Oxrr,'b')
xlabel('Time(s)'); ylabel('Position (m)')
legend('Rear Bump Input','Rear Suspension Displacement')
%FFT
subplot(2,1,1)
plot(t,Oxlr)
L=length(t);
Fs=length(t)/t(length(t));
NFFT = 2^nextpow2(L); % Next power of 2 from length of y
Y = fft(Oxlr,NFFT)/L;
f = Fs/2*linspace(0,1,NFFT/2+1);
ylabel('Suspension Displacement (m)')
xlabel('Time (s)')
subplot(2,1,2)
% Plot single-sided amplitude spectrum.
plot(f,2*abs(Y(1:NFFT/2+1)))
xlabel('Frequency (Hz)')
ylabel('$|Y(f)|$ (Fast Fourier Transform)')